Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Solids and Structures 42 (2005) 4880–4889

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

On dynamic stress analysis for cracks in elastic materials with voids

G. Iovane [a,*], M.A. Sumbatyan [b]

[a] *D.I.I.M.A., University of Salerno, Via Ponte don Melillo, 84084 Fisciano (SA), Italy*
[b] *Faculty of Mechanics and Mathematics, Rostov State University, Zorge Street 5, Rostov-on-Don 344090, Russia*

Available online 9 March 2005

Abstract

The present paper is concerned with the development of a semi-analytical approach to the dynamic problem of the concentration of stresses near the edges of a crack located in a porous elastic space (two-dimensional problem) and subjected to a normal oscillating load applied to the crack faces. Our analysis is made in the context of the Goodman–Cowin–Nunziato (G–C–N) theory for porous media. In previous work we studied static crack problems for such materials; now we introduce an analysis of the relevant dynamic aspects. By using the Fourier transform, the problem is reduced in explicit form to a hyper-singular integral equation with a convolution kernel valid over the crack length. Then, we apply a collocation technique developed in our previous work to solve this equation, and study the stress intensity factor. The principal goal is to compare the stress intensity factor for the static and dynamic cases. We also compare our results with the case of an ordinary linear elastic medium.

1. Introduction

The investigation of the dynamical properties of various cracked elastic solids is an important problem in the practice of ultrasonic inspection of materials, vibrations of engineering structures on elastic foundations, in soil mechanics, seismology and many other fields. Usually, the materials mentioned above can be correctly described by dynamic equations of classical linear isotropic elastic solids (Achenbach, 1980). However such materials as soils, composite materials, granular materials, etc., show a specific characteristic response to the applied dynamic load.

* Corresponding author.
E-mail addresses: iovane@diima.unisa.it (G. Iovane), sumbat@math.rsu.ru (M.A. Sumbatyan).

0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2005.01.020

There is a number of appropriate theories describing dynamic properties of porous materials, and the most classical one is certainly a Biot consolidation theory of fluid-saturated porous solids (Biot, 1956; Biot and Willis, 1957). Typically, these theories reduce to ordinary elasticity when the pore fluid is absent. However, many real materials possess porosity as a dry porous substance, hence Biot's theory cannot describe mechanical properties of such voided media. That is why Goodman, Cowin and Nunziato proposed a new theory, which was developed in detail both for linear and nonlinear materials (Goodman and Cowin, 1972; Cowin and Nunziato, 1983). This theory is applicable to dry media too. Some static crack problems for such materials have been studied in our recent works (Iovane et al., 2003; Ciarletta et al., 2003). Puri and Cowin (1985) studied various types of plane waves, which can propagate in these voided linear elastic materials. Recently, Chandrasekharaiah (1987) studied a propagation of the Rayleigh surface waves in such porous media.

In the present work we develop a semi-analytical approach to a dynamic problem concerning concentration of stresses near the edges of a crack located in the porous elastic space (two-dimensional problem) and subjected to a normal oscillating load applied to the crack faces. As known from the literature, in the classical static problem, if a normal load is applied to the faces of the crack, then the shape of the faces near its edge under this stress can be represented explicitly as a root-square function. This permits an analytical calculation of the stress intensity factor in the classical case. Obviously, stress analysis of the porous materials is very important in the engineering practice. Some previous results of these authors, carried out in the static case, show that the stress intensity factor decreases when compared with the case of ordinary elasticity. The question, whether this property is valid in the dynamic problem, is not so simple. This requires rather refined numerical analysis, that is one of the objectives of the present study.

By using the Fourier transform, the problem is reduced in explicit form to a hyper-singular integral equation with a convolution kernel valid over the crack length. Then we apply a collocation technique developed in our previous works to solve this equation, and study the stress intensity factor. The principal goal is to compare the stress intensity factor in the static and dynamic cases. We also compare our results with the case of ordinary linear elastic medium.

The paper is organized as follows. In Section 2 we give a survey of some previous results; in Section 3 we transform the problem to a certain hyper-singular integral equation; while, in Section 4 we study the properties of the kernel of this hyper-singular integral equation and propose a direct numerical approach to solve it; in Section 5 we formulate the physical conclusions and give a discussion of the obtained results.

2. Survey of previous results

The theory of linear isotropic elastic materials with voids, in the case of harmonic oscillations when the dependence on time is taken in the form $\exp(-\mathrm{i}\Omega t)$, can be described by the following equations of motion (Puri and Cowin, 1985; Chandrasekharaiah, 1987):

$$\mu\Delta\bar{u} + (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\bar{u} + \beta\,\mathrm{grad}\,\phi + \rho\Omega^2\bar{u} = 0, \tag{2.1a}$$

$$\alpha\Delta\phi - \xi\phi - \beta\,\mathrm{div}\,\bar{u} + (\mathrm{i}\omega\Omega + \rho k\Omega^2)\phi = 0. \tag{2.1b}$$

Here Ω is the angular frequency, $\bar{u} = \{u_1, u_2, u_3\}$ is the displacement vector; $\phi = v - v_0$, the change in the volume fraction from the reference volume fraction; λ and μ, classical elastic moduli; ρ, the mass density of the material; Δ, the Laplacian; α, β, ξ, ω, k, material coefficients related to the porosity. The harmonic time factor $\exp(-\mathrm{i}\Omega t)$, which is present in front of all physical quantities, is omitted throughout the paper.

It is obvious that in the case $\beta = 0$ the elastic displacement field $\bar{u}$ and the "porosity" field ϕ can be separately determined from Eqs. (2.1a) and (2.1b), respectively.

The components of the stress tensor can be defined in terms of functions $\bar{u}$ and ϕ from the constitutive equations (Cowin and Nunziato, 1983) (δ_{ij} is Kronecker's delta)

$$\sigma_{ij} = \lambda\delta_{ij}\varepsilon_{kk} + 2\mu\varepsilon_{ij} + \beta\phi\delta_{ij}, \qquad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{2.2}$$

with summation on repeating index.

It can be easily seen that the so-called "anti-plane" problem, where the displacement field is

$$\bar{u} = \{0, 0, w(x, y)\}, \qquad \phi = 0, \tag{2.3}$$

can be trivially reduced to the classical case concerning the waves of horizontal polarization, which is described by the single standard Helmholtz equation, containing the transverse wave number k_s:

$$\Delta w + k_s^2 w = 0, \qquad k_s = \frac{\Omega}{c_s}, \qquad c_s^2 = \frac{\mu}{\rho}. \tag{2.4}$$

So here we study only the two-dimensional problem in the case of vertical polarization (the so-called "in-plane" problem):

$$\bar{u} = \{u(x, y), v(x, y), 0\}, \qquad \phi = \phi(x, y), \tag{2.5}$$

for which Eqs. (2.1) become

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} + c^2\dfrac{\partial^2 u}{\partial y^2} + (1 - c^2)\dfrac{\partial^2 v}{\partial x \partial y} + H\dfrac{\partial \phi}{\partial x} + k_p^2 u = 0, \\ \dfrac{\partial^2 v}{\partial y^2} + c^2\dfrac{\partial^2 v}{\partial x^2} + (1 - c^2)\dfrac{\partial^2 u}{\partial x \partial y} + H\dfrac{\partial \phi}{\partial y} + k_p^2 v = 0, \\ \dfrac{\partial^2 \phi}{\partial x^2} + \dfrac{\partial^2 \phi}{\partial y^2} + \left(\dfrac{\mathrm{i}\omega}{\alpha}\Omega + \dfrac{\rho k}{\alpha}\Omega^2 - \dfrac{\xi}{\alpha}\right)\phi - \dfrac{\beta}{\alpha}\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = 0. \end{cases} \tag{2.6}$$

For all that the (non-trivial) components of the stress tensor are expressed in terms of components of the displacement vector as follows:

$$\begin{aligned} \frac{\sigma_{xx}}{\lambda + 2\mu} &= \frac{\partial u}{\partial x} + (1 - 2c^2)\frac{\partial v}{\partial y} + H\phi, \\ \frac{\sigma_{yy}}{\lambda + 2\mu} &= (1 - 2c^2)\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + H\phi, \\ \frac{\sigma_{xy}}{\mu} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}. \end{aligned} \tag{2.7}$$

Here in formulas (2.6), (2.7)

$$k_p = \frac{\Omega}{c_p}, \quad c_p^2 = \frac{\lambda + 2\mu}{\rho}, \quad H = \frac{\beta}{\lambda + 2\mu}, \quad c^2 = \frac{c_s^2}{c_p^2} = \frac{k_p^2}{k_s^2} < 1, \tag{2.8}$$

and c_p is the well known longitudinal wave speed.

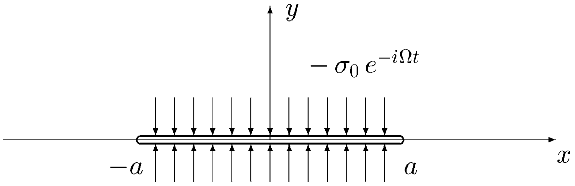

Fig. 1. A linear crack dislocated in a porous elastic plane, under a uniform harmonically oscillating normal load.

Let a linear crack of the length $2a$ be located over the line $y = 0$ on the interval $|x| < a$ (see Fig. 1). Let an oscillating normal load of the amplitude $-\sigma_0$ be symmetrically applied to the faces of the crack and there is no load at infinity. Then, due to the natural symmetry, we can consider only the upper half-plane $y \geqslant 0$, with the following boundary conditions over the line $y = 0$:

$$\sigma_{xy} = 0, \quad \frac{\partial \phi}{\partial y} = 0 \ \ (|\, x \,| < \infty), \quad \sigma_{yy} = -\sigma_0 \ \ (|\, x \,| < a), \quad v = 0 \ \ (|\, x \,| > a). \tag{2.9}$$

It is proved (Atkin et al., 1990) that the boundary condition for the function ϕ (2.9) directly follows from the principle of energy balance.

Chandrasekharaiah (1987) has shown that Eqs. (2.6) are automatically satisfied if the classical wave potentials, p and q

$$u = \frac{\partial p}{\partial x} - \frac{\partial q}{\partial y}, \qquad v = \frac{\partial p}{\partial y} + \frac{\partial q}{\partial x} \tag{2.10}$$

satisfy the following equations:

$$\Delta q + k_s^2 q = 0, \tag{2.11}$$

$$\left[\left(\Delta + k_p^2\right)\left(\Delta - \frac{1}{l_2^2} + \frac{\mathrm{i}\omega}{\alpha}\Omega + \frac{\rho k}{\alpha}\Omega^2\right) + \frac{H}{l_1^2}\Delta\right] p = 0, \tag{2.12}$$

where $l_1^2 = \alpha/\beta,\ l_2^2 = \alpha/\xi$ are some physical parameters of dimension of length. Then the function ϕ can be determined from the equation

$$-H\phi = \Delta p + k_p^2 p. \tag{2.13}$$

Let us introduce the new unknown function $g(x)$ as follows:

$$v(x, 0) = \begin{cases} g(x), & |\, x \,| < a, \\ 0, & |\, x \,| > a. \end{cases} \tag{2.14}$$

Obviously, boundary conditions (2.9) in terms of wave potentials p and q, with the use of (2.10), can be written in the form $(y = 0, |x| < \infty)$

$$\sigma_{xy} = 0 \Rightarrow 2\frac{\partial^2 p}{\partial x \partial y} + \frac{\partial^2 q}{\partial x^2} - \frac{\partial^2 q}{\partial y^2} = 0, \tag{2.15a}$$

$$\frac{\partial \phi}{\partial y} = 0 \Rightarrow \frac{\partial}{\partial y}(\Delta p + k_p^2 p) = 0, \tag{2.15b}$$

$$v = g(x) \Rightarrow \frac{\partial p}{\partial y} + \frac{\partial q}{\partial x} = g(x). \tag{2.15c}$$

3. Reducing the problem to integral equation

Let us apply the Fourier transform along x-axis to Eqs. (2.11), (2.12) and to boundary conditions (2.15), that is to establish a correspondence between the original of any given function $f(x, y)$ and its Fourier image $F(s, y)$:

$$F(s,y)=\int_{-\infty}^{\infty}f(x,y)\mathrm{e}^{\mathrm{i}sx}\,\mathrm{d}x,\qquad f(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}F(s,y)\mathrm{e}^{-\mathrm{i}sx}\,\mathrm{d}s. \tag{3.1}$$

Obviously, any derivative $\partial/\partial x$ with respect to x is equivalent in Fourier images to a multiplication by the factor $(-\mathrm{i}s)$, and the second-order derivative $\partial^2/\partial x^2$ involves the factor $(-s^2)$. Then any partial derivative with respect to y transforms to a certain ordinary derivative with some parameter s. Note that the Fourier transform of any function is designated here by corresponding capital letter.

Eqs. (2.11), (2.12) in terms of the Fourier images are

$$Q''+(k_s^2-s^2)Q=0, \tag{3.2}$$

$$\left[\left(k_p^2+\frac{\mathrm{d}^2}{\mathrm{d}y^2}-s^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2}-s^2-\frac{1}{l_2^2}+\frac{\mathrm{i}\omega}{\alpha}\Omega+\frac{\rho k}{\alpha}\Omega^2\right)+\frac{H}{l_1^2}\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2}-s^2\right)\right]P=0, \tag{3.3}$$

and boundary conditions (2.15) ($y=0$) are respectively:

$$2(-\mathrm{i}s)P'-s^2Q-Q''=0, \tag{3.4a}$$

$$\left(-s^2P+P''\right)'+k_p^2P'=0, \tag{3.4b}$$

$$P'-\mathrm{i}sQ=G(s),\qquad G(s)=\int_{-a}^{a}g(\tau)\exp(\mathrm{i}s\tau)\,\mathrm{d}\tau, \tag{3.4c}$$

where all ordinary derivatives are applied with respect to variable y, and $G(s)$ is the Fourier transform of function $g(x)$.

Eqs. (3.2) and (3.3) are ordinary differential equations with constant coefficients; so the well known method of characteristic polynomials can be used to construct their solution. Its application to Eq. (3.2) gives

$$Q(s,y)=D(s)\mathrm{e}^{-\gamma_3(s)y},\qquad \gamma_3(s)=\sqrt{s^2-k_s^2}, \tag{3.5}$$

where $D(s)$ is an unknown coefficient. It should be noted, when considering two possible roots of the characteristic second-order algebraic equation, that only the root $-\gamma_3(s)$ satisfies Sommerfeld's radiation condition. Of course, the principal value of the root-square is taken so that $Re[\sqrt{s^2-k_s^2}]\geqslant 0$.

The characteristic equation for Eq. (3.3) is directly obtained, if one seeks its solution in the form $P=A\exp(\chi y)$, that leads to

$$(k_p^2+\chi^2-s^2)[l_2^2(\chi^2-s^2)-1+\mathrm{i}\omega^*k_p+k^{*2}k_p^2]+N(\chi^2-s^2)=0, \tag{3.6}$$

or

$$l_2^2(s^2-\chi^2)^2+[1-N-\mathrm{i}\omega^*k_p-(l_2^2+k^{*2})k_p^2](s^2-\chi^2)-k_p^2(1-\mathrm{i}\omega^*k_p-k^{*2}k_p^2)=0, \tag{3.7}$$

where $0<N=(l_2^2/l_1^2)H<1$ is a dimensionless parameter, and the coefficients $\omega^*=\omega l_2^2c_p/\alpha$ and $k^*=l_2c_p\sqrt{\rho k/\alpha}$ are of dimension of length. The last quadratic equation (3.7) gives for the parameter $k_{1,2}^2=s^2-\chi_{1,2}^2$ the following values:

$$k_{1,2}^2=\frac{1}{2l_2^2}\left\{-[1-N-\mathrm{i}\omega^*k_p-(l_2^2+k^{*2})k_p^2]\pm\sqrt{[1-N-\mathrm{i}\omega^*k_p-(l_2^2+k^{*2})k_p^2]^2+4l_2^2k_p^2(1-\mathrm{i}\omega^*k_p-k^{*2}k_p^2)}\right\}, \tag{3.8}$$

that leads to the following representation for the function P:

$$P(s,y) = A(s)e^{-\gamma_1(s)y} + B(s)e^{-\gamma_2(s)y}, \qquad \gamma_{1,2}(s) = \sqrt{s^2 - k_{1,2}^2}, \tag{3.9}$$

in accordance with Sommerfeld's principle, if one takes again the principal value of the root-square.

Now the substitution of functions Q (3.5) and P (3.9) into boundary conditions (3.4) leads to the following system of linear algebraic equations with respect to the coefficients A, B, D

$$\begin{cases} 2is\gamma_1 A + 2is\gamma_2 B - (\gamma_3^2 + s^2)D = 0, \\ \gamma_1(k_p^2 - k_1^2)A + \gamma_2(k_p^2 - k_2^2)B = 0, \\ -\gamma_1 A - \gamma_2 B - isD = G(s), \end{cases} \tag{3.10}$$

whose solution is

$$A(s) = \frac{\gamma_3^2 + s^2}{\gamma_1 k_s^2}\frac{k_p^2 - k_2^2}{k_1^2 - k_2^2}G(s), \quad B(s) = \frac{\gamma_3^2 + s^2}{\gamma_2 k_s^2}\frac{k_p^2 - k_1^2}{k_2^2 - k_1^2}G(s), \quad D(s) = \frac{2is}{k_s^2}G(s). \tag{3.11}$$

Further, in order to apply the last two boundary conditions in Eq. (2.9), we should write out the expression for the stress σ_{yy}, which in terms of wave potentials is (see Eqs. (2.7) and (2.10))

$$\frac{\sigma_{yy}}{\rho c_s^2} = k_s^2 p + 2\left(\frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 q}{\partial x \partial y}\right) \Rightarrow \frac{\Sigma_{yy}(s,0)}{\rho c_s^2} = k_s^2 P + 2\left(isQ' - s^2 P\right)$$
$$= (k_s^2 - 2s^2)[A(s) + B(s)] - 2is\gamma_3 D(s). \tag{3.12}$$

By using Eq. (3.11), this leads to the following expression for the Fourier transform of the (complex-valued) amplitude of the normal load over the crack faces, in terms of function $G(s)$:

$$\frac{\Sigma_{yy}(s,0)}{\rho c_s^2} = -L(s)G(s),$$
$$L(s) = \frac{(2s^2 - k_s^2)^2[(k_p^2 - k_2^2)/\gamma_1 - (k_p^2 - k_1^2)/\gamma_2] - 4(k_1^2 - k_2^2)s^2\gamma_3}{k_s^2(k_1^2 - k_2^2)}. \tag{3.13}$$

Now, by applying the inverse Fourier transform, with the use of the convolution theorem, and taking into account that the normal stress $\sigma_{yy}(x,0)$, $|x| \leqslant a$, is known (see boundary conditions (2.9)), we come to the integral equation

$$\int_{-a}^{a} g(\tau)K(x-\tau)\,d\tau = \frac{\sigma_0}{\rho c_s^2}, \quad |x| \leqslant a,$$
$$K(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} L(s)e^{-isx}\,ds = \frac{1}{\pi}\int_0^{\infty} L(s)\cos(sx)\,ds. \tag{3.14}$$

It can directly be shown that in the case of classical elasticity when $N = \beta = 0$, we have $k_1^2 = k_p^2$, $k_2^2 = -(1 - i\omega^* k_p - k^{*2}k_p^2)/l_2^2$, and the Fourier transform of kernel (3.13), function $L(s)$ (the so-called "symbolic function"), reduces to the well known form

$$L(s) = \frac{(2s^2 - k_s^2)^2 - 4s^2\gamma_1\gamma_3}{k_s^2\gamma_1}, \tag{3.15}$$

containing the classical Rayleigh function in the numerator. This serves as an additional control of correctness the basic representation (3.13), (3.14).

4. Some properties of the kernel and numerical treatment

Let us first note that

$$\frac{1}{\gamma_n}=\frac{1}{s}+\frac{k_n^2}{2s^3}+\frac{3k_n^4}{8s^5}+O\left(\frac{1}{s^7}\right),\quad (n=1,2),\qquad s\to\infty,$$
$$\gamma_3=s-\frac{k_s^2}{2s}-\frac{k_s^4}{8s^3}+O\left(\frac{1}{s^5}\right),\qquad s\to\infty,\tag{4.1}$$

consequently

$$L(s)=As+\frac{B}{s}+O\left(\frac{1}{s^3}\right),\qquad s\to\infty,$$
$$A=2\frac{k_p^2-k_s^2}{k_s^2},\qquad B=\frac{3}{2}k_s^2-2k_p^2+\frac{3}{2k_s^2}\left[k_p^2(k_1^2+k_2^2)-k_1^2k_2^2\right].\tag{4.2}$$

Let us study the qualitative properties of the kernel $K(x)$. It is clear that integral (3.14) representing the kernel does not converge at infinity in any classical sense, since the integrand is unbounded as $s\to\infty$. However, this can be treated as a generalized function (see Gel'fand and Shilov, 1964). Namely,

$$K(x)=\frac{1}{\pi}\int_0^\infty L(s)\cos sx\,ds=\frac{A}{\pi}\int_0^\infty s\cos sx\,ds+\frac{B}{\pi}\int_0^\infty\frac{\cos sx-e^{-s}}{s}ds+K_0(x)$$
$$=\frac{A}{\pi}\lim_{\varepsilon\to+0}\int_0^\infty e^{-\varepsilon s}s\cos(sx)\,ds-\frac{B}{\pi}\ln|x|+K_0(x)=\frac{A}{\pi}\lim_{\varepsilon\to+0}\frac{\varepsilon^2-x^2}{(\varepsilon^2+x^2)^2}-\frac{B}{\pi}\ln|x|+K_0(x)$$
$$=-\frac{A}{\pi x^2}-\frac{B}{\pi}\ln|x|+K_0(x),\tag{4.3}$$

where $K_0(x)$ is a regular (at least differentiable) kernel:

$$K_0(x)=\frac{1}{\pi}\int_0^\infty\left\{[L(s)-As]\cos(sx)-B\frac{\cos(sx)-e^{-s}}{s}\right\}ds.\tag{4.4}$$

Here we have used the table integral (Gradshteyn and Ryzhik, 1994)

$$\int_0^\infty\frac{\cos sx-e^{-s}}{s}ds=-\ln|x|.\tag{4.5}$$

Representation (4.3) shows that the kernel of integral equation (3.14), function $K(x)$, is hyper-singular. An adequate treatment of hyper-singular integrals and an efficient numerical method to solve such integral equations can be found in our previous papers (Iovane et al., 2003; Ciarletta et al., 2003). Briefly speaking, our numerical approach can be described as follows.

First of all, we divide the interval $(-a,a)$ to n small equal subintervals of the length $h=2a/n$, by the nodes $-a=t_0,t_1,t_2,\dots,t_{n-1},t_n=a,t_j=-a+jh,\quad j=0,1,\dots,n$. The central point of each sub-interval (t_{i-1},t_i) is denoted as x_i, hence $x_i=-a+(i-1/2)h, i=1,\dots,n$. Then we prove that a correct discrete approximation to hyper-singular, logarithmic and regular parts of the kernel is given (with a small h) by the following formulas:

$$\int_{-a}^{a}\frac{g(\tau)\,d\tau}{(x_i-\tau)^2}\approx\sum_{j=1}^{n}g(t_j)\left(\frac{1}{x_i-t_j}-\frac{1}{x_i-t_{j-1}}\right),$$
$$\int_{-a}^{a}g(\tau)\ln|x_i-\tau|\,d\tau\approx\sum_{j=1}^{n}g(t_j)\{|x_i-t_j|\left[\ln|x_i-t_j|-1\right]-|x_i-t_{j-1}|\left[\ln|x_i-t_{j-1}|-1\right]\},$$
$$\int_{-a}^{a}g(\tau)K_0(x_i-\tau)\,d\tau\approx h\sum_{j=1}^{n}g(t_j)K_0(x_i-t_j).\tag{4.6}$$

If one substitutes these approximate expressions to the basic integral Eq. (3.14), with the use of Eq. (4.3), one comes to a regular linear algebraic $n \times n$ system with respect to the values $g_j = g(t_j)$ of the unknown function $g(x)$ at the nodes. The convergence of such an approach to a bounded solution of the integral equation as $n \to \infty$, or $h \to 0$, is proved in (Iovane et al., 2003).

Some examples of the numerical calculations are demonstrated in Figs. 2–4. Here we determine numerically the stress intensity factor K near the tip of the crack, which can be calculated as follows (see Ciarletta et al., 2003):

$$K \sim \lim_{x \to a+0} \mid \sigma_{yy}(x,0) \mid \sqrt{x^2 - a^2} \sim \lim_{x \to a+0} \left| \int_{-a}^{a} \frac{g(\tau)\,d\tau}{(x-\tau)^2} \right| \sqrt{x^2 - a^2}. \tag{4.7}$$

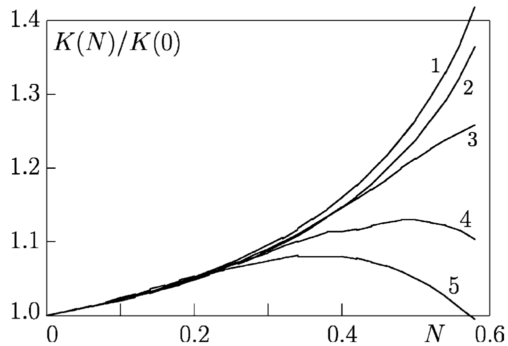

Fig. 2. Relative value of the stress intensity factor K to the one in the classical elastic medium ($N = 0$) versus the coupling number N: $c^2 = c_s^2/c_p^2 = k_p^2/k_s^2 = 0.4$; $a/l_2 = 3$; $\omega^*/a = 0.5$; $k^*/a = 0.8$. Line 1: $ak_s = 0$, line 2: $ak_s = 0.2$, line 3: $ak_s = 0.6$, line 4: $ak_s = 0.8$, line 5: $ak_s = 1.0$.

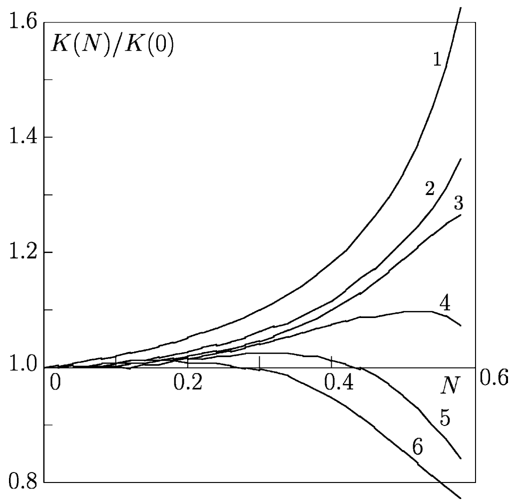

Fig. 3. Relative value of the stress intensity factor K to the one in the classical elastic medium ($N = 0$) versus the coupling number N: $c^2 = c_s^2/c_p^2 = k_p^2/k_s^2 = 0.4$; $a/l_2 = 6$; $\omega^*/a = 0.5$; $k^*/a = 0.8$. Line 1: $ak_s = 0$, line 2: $ak_s = 0.2$, line 3: $ak_s = 0.4$, line 4: $ak_s = 0.6$, line 5: $ak_s = 0.8$, line 6: $ak_s = 1.0$.

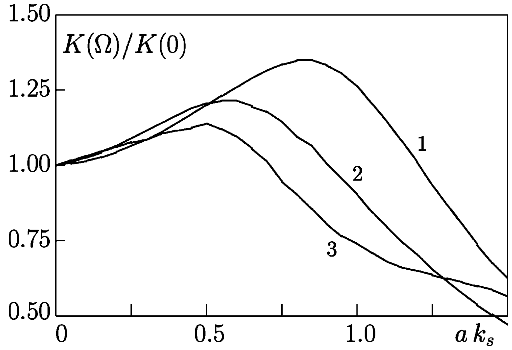

Fig. 4. Relative value of the stress intensity factor K to the one in the static case ($\Omega = 0$) versus frequency parameter ak_s: $c^2 = c_s^2/c_p^2 = k_p^2/k_s^2 = 0.4$; $N = 0.5$; $\omega^*/a = 0.5$; $k^*/a = 0.8$. Line 1: $a/l_2 = 1$, line 2: $a/l_2 = 3$, line 3: $a/l_2 = 6$.

5. Physical conclusions and discussions

We study the dynamic behaviour of porous elastic materials, in frames of the Goodman–Cowin–Nunziato (G–C–N) model. This model contains a number of physical parameters, with a part of them being classical elastic moduli whose mechanical meaning and typical values are quite clear. Thus, the quantities c_p, c_s and k_p, k_s are the classical longitudinal and transverse wave speeds and wave numbers, respectively, hence the ratio $c^2 = c_s^2/c_p^2 = k_p^2/k_s^2$ is a certain (classical) positive quantity less than 1. Then, parameter l_2 indicates a certain intrinsic size of the porous medium, certainly this is related to an average size of the voids. Hence, the (dimensionless) ratio a/l_2 determines the relative length of the crack compared with this parameter l_2. The coupling number, N is a positive quantity, whose value is also less than 1. The closer this value to 1 the higher the porosity of the material.

The G–C–N theory of porous elastic materials is well developed in the general context (see, for example, Goodman and Cowin, 1972; Cowin and Nunziato, 1983; Atkin et al., 1990). Particularly, some important energetic properties of such materials, as well as the existence and uniqueness of many problems in the linear case have been substantiated . But in the context of possible applications there are too many open questions, and the most important of them is of cause the question concerning the values of some intrinsic physical parameters. For instance, the physical meaning of two parameters ω^*, k^* (both of them—of dimension of length), which arise only in the dynamic processes, are less clear in the present G–C–N model, so we accept their values as for some hypothetical voided elastic media. One can see from Figs. 2–4, in which degree these parameters influence the stress intensity factor.

An interesting question is also related to the value of the stress intensity factor K, when compared with the case of static problem and the case of ordinary linear elastic material. In our previous work (Ciarletta et al., 2003) we showed numerically that the stress intensity factor always grows with increasing of the coupling number N in the static case (compare also with Figs. 2 and 3, line 1). By other words, in the static case the stress concentration near the tip in the porous material is always higher than the one in the ordinary elastic material. Here in the dynamic case we can see from Figs. 2 and 3 that the behaviour of this factor becomes more intricate, and our calculations show that in some cases the stress intensity factor in porous dynamics can be higher than respective value in the classical material, but in some other cases this decreases with the porosity increasing. It is not so easy to predict this behaviour a priori, and the exact numerical simulation only allows us to correctly predict this behaviour.

Another interesting aspect is connected with the question concerning the variation of factor K versus frequency, when the latter becomes relatively high. Numerous numerical experiments carried out show that in the low frequency range the frequency increasing results in higher values of the stress intensity factor (a typical example is demonstrated in Fig. 4), but with further increase of the frequency this factor decreases.

At last, we would like to outline the regimes when the stress intensity factor attains its maximum values that can be very important in applications. Our detailed numerical investigation shows that the maximum (and so, more dangerous) values of the stress intensity factor near the tip of the crack are attained for lower values of the porosity (i.e. lower values of N) and for moderate values of the transverse frequency parameter ak_s (approximately, around the value $ak_s \sim 1$). A typical example of calculations is demonstrated in Fig. 4.

Acknowledgment

The paper has been supported by GNFM of Italian INDAM. The paper has also been supported in part by the Russian President Program for leading scientific schools in mechanics (Grant 2113.2003.1).

References

Achenbach, J.D., 1980. Wave Propagation in Elastic Solids. North-Holland, Amsterdam.

Atkin, R.J., Cowin, S.C., Fox, N., 1990. On boundary conditions for polar materials. J. Acoust. Soc. Am. 87, 2349–2358.

Biot, M.A., 1956. Theory of propagation of elastic waves in a fluid-saturated porous solid. I: Low frequency range. J. Acoust. Soc. Am. 28, 168–178.

Biot, M.A., Willis, D.G., 1957. Elastic coefficients of the theory of consolidation. J. Appl. Mech. 24, 594–601.

Chandrasekharaiah, D.S., 1987. Effects of surface stresses and voids on Rayleigh waves in an elastic solid. Int. J. Eng. Sci. 25, 205–211.

Ciarletta, M., Iovane, G., Sumbatyan, M.A., 2003. On stress analysis for cracks in elastic materials with voids. Int. J. Eng. Sci. 41, 2447–2461.

Cowin, S.C., Nunziato, J.W., 1983. Linear elastic materials with voids. J. Elasticity 13, 125–147.

Gel'fand, I.M., Shilov, G.E., 1964. Generalized Functions, Vol. 1. Academic Press, London.

Goodman, M.A., Cowin, S.C., 1972. A continuum theory for granular materials. Arch. Rat. Mech. Anal. 44, 249–266.

Gradshteyn, I.S., Ryzhik, I.M., 1994. Table of Integrals, Series, and Products, 5th ed. Academic Press, New York.

Iovane, G., Lifanov, I.K., Sumbatyan, M.A., 2003. On direct numerical treatment of hypersingular integral equations arising in mechanics and acoustics. Acta Mech. 162, 99–110.

Puri, P., Cowin, S.C., 1985. Plane waves in linear elastic materials with voids. J. Elasticity 15, 167–183.